# Interacting Network Elements: Chaos and Congestion Propagation

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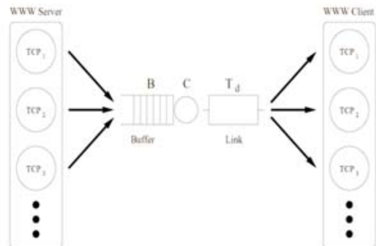
# Outline

- The more complex word of ad-hoc, mobile networks and p2p applications
- Complex system approach
- Chaotic and non-linear dynamics
- Interaction of processes and traffic flows
- Pattern formation in traffic, congestion propagation

# The new more complex word …

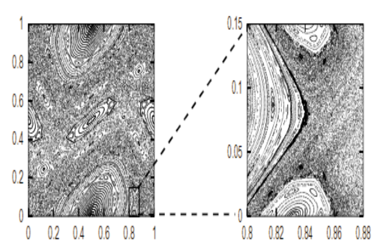
- P2P traffic vs. web traffic → hosts are more equal, core/access distinction is not that important
- Mobile, ad-hoc: bandwidth is fully utilized
- Strong interaction






# Complexity and non-linear research

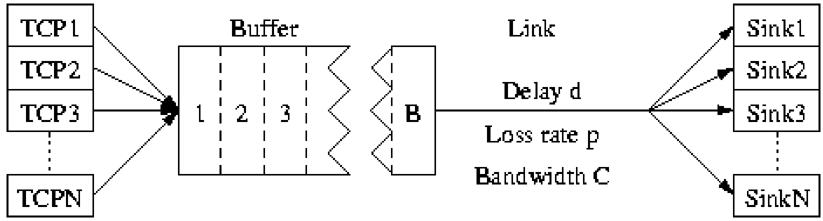
- Complexity research deals with large number of strongly interacting systems, (IST-FET-Complexity)
- Chaos, fractals/scaling pattern formation, granular and vehicular traffic…



# Chaos



# Simplest network model



# Periodicity

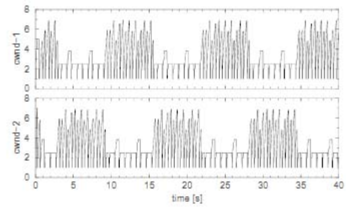



Fig. 4. The congestion window processes of two competing TCP sources. $C = 0.5$ Mbps, $d = 10$ ms, $B = 4$ packets.

Fig. 5. Spatio-temporal graph: $C = 0.5$ Mbps, $d = 10$ ms, $B = 4$ packets (the same configuration was used as for Fig 4).

Veres & Boda INFOCOM 2000



# Chaos

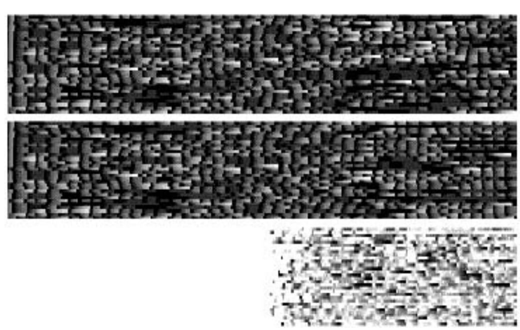


Fig. 9. Spatio-temporal graph of the original system (top). Spatio-temporal graph of the perturbed system (middle). Difference between the two systems (bottom).

Veres & Boda INFOCOM 2000

# Lyapunov properties

$$E(t) = \sqrt{\sum_{i=1}^{N}(w^{orig}(i,t) - w^{pert}(i,t))^2}.$$

$$\lambda(t_0, i) \approx \frac{1}{\Delta t} \ln \left| \frac{E(t_0 + \Delta t)}{\epsilon_i} \right|$$

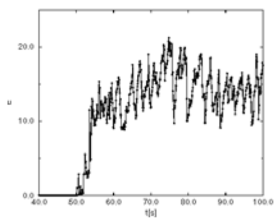


Fig. 10. Divergence of the original and the perturbed systems.

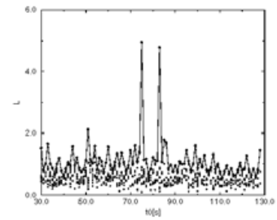


Fig. 11. Lyapunov exponents at different points of the trajectory and for all 30 TCPs, maximum exponents along the trajectory are connected with a line (average maximum $\lambda \approx 1.11$).

Veres & Boda INFOCOM 2000



# Strange attractor

$$x[i] = \frac{1}{n}\sum_{j=1}^{n} \text{cwnd}_x[i-j]$$

$$y[i] = \frac{1}{n}\sum_{j=1}^{n} \text{cwnd}_y[i-j]$$

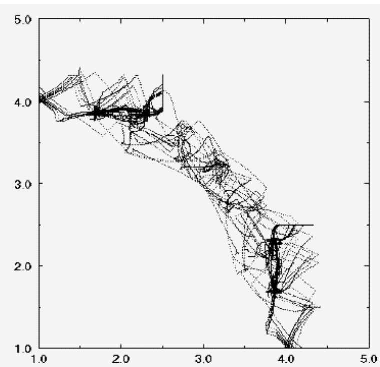
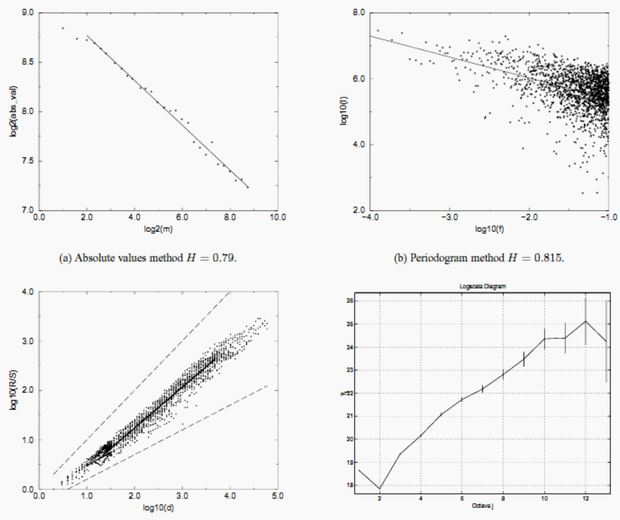



Fig. 7. Strange attractor. $C = 0.1$ Mbps, $d = 10$ ms, $B = 4$ packets. a) time shifts $n = 100$, b) time shifts $n = 300$.

Veres & Boda INFOCOM 2000




(a) Absolute values method $H = 0.79$.

(b) Periodogram method $H = 0.815$.

(c) R/S method $H = 0.813$.

(d) Wavelet method $H = 0.787$, $(0.754, 0.819)$.

Fig. 13. LRD tests for one-TCP microflow.

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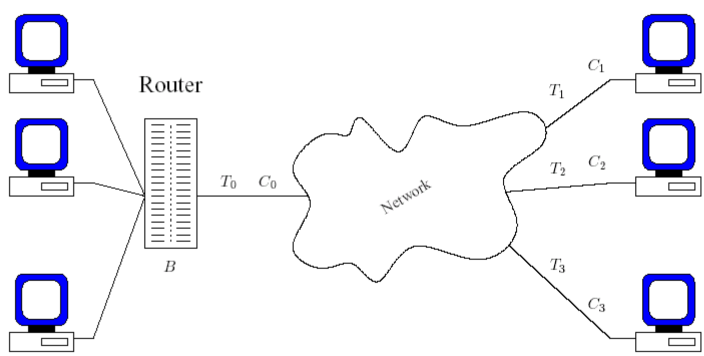
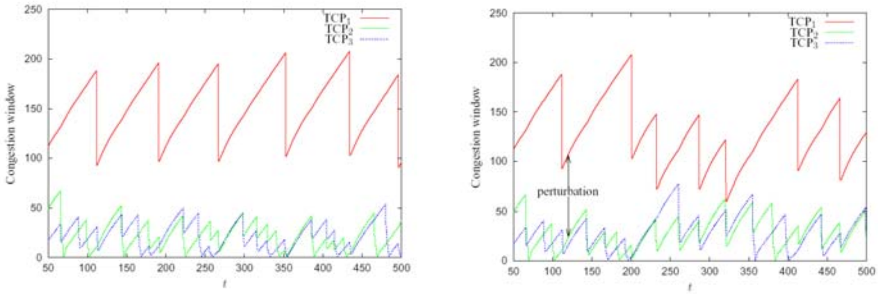
# 3 TCPs with different round trip times



# Congestion window evolution




(a) (b)



# Differential equations

$$\frac{dw_i}{dt} = \frac{1}{T_0 + T_i + bP/C_0}, \qquad (22)$$

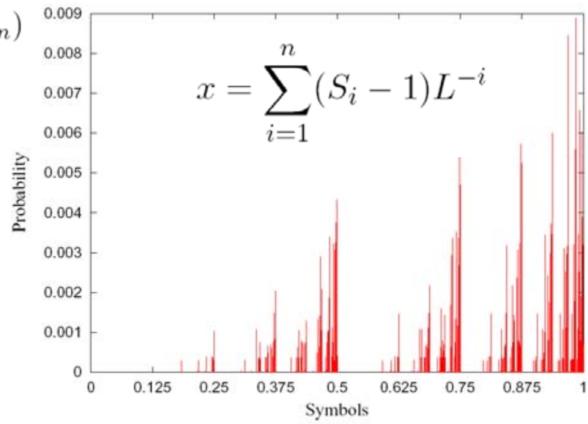
where $b$ is the actual number of packets in the buffer. To keep things simple, we estimate the number of packets in the system by the sum of congestion windows and the packets traveling in the lines by $C_0T_0/P + \frac{1}{3}C_0(T_1+T_2+T_3)/P$. This way the queue length in the buffer is estimated as

$$b = \sum_i w_i - C_0T_0/P + \frac{1}{3}C_0(T_1+T_2+T_3)/P.$$

# Symbol sequence probability

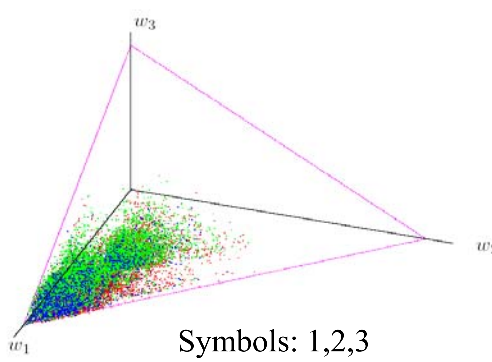
$$P(S_1, S_2, \ldots, S_n)$$

$$S_i = \{1, 2, \ldots, L\}$$



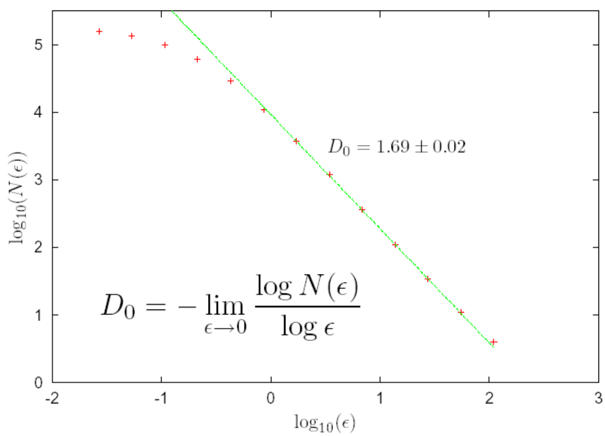



# Poincarè surface of section



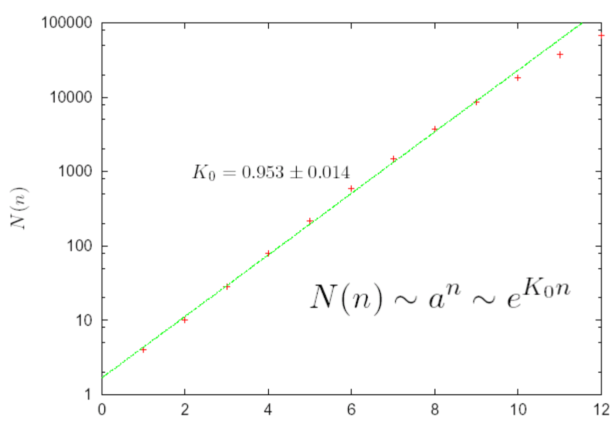

Symbols: 1,2,3

# Fractal dimension of the attractor






# Topological entropy

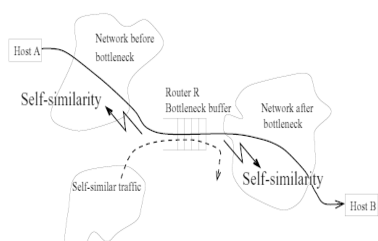


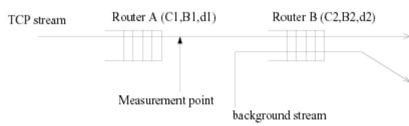
# Interaction of flows



# Interacting traffic flows

- Traffic flows crossing the same bottleneck can inherit scaling properties from each other





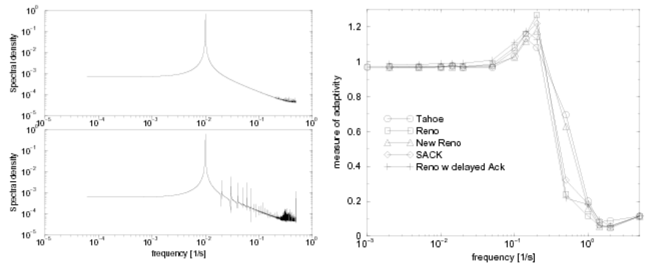
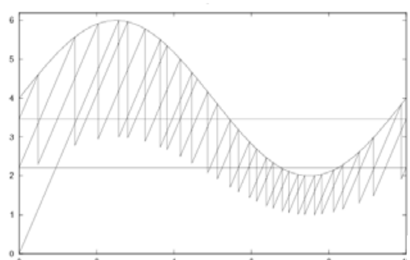




Fig. 4. a) Frequency response to a sine wave of $f = 0.01[1/s]$ (top: background sine wave, bottom: TCP response). In this configuration the measure of adaptivity is $D(0.01) \approx 1$. b) Measure of adaptivity $D(f)$ as a function of the frequency for several TCP variants.

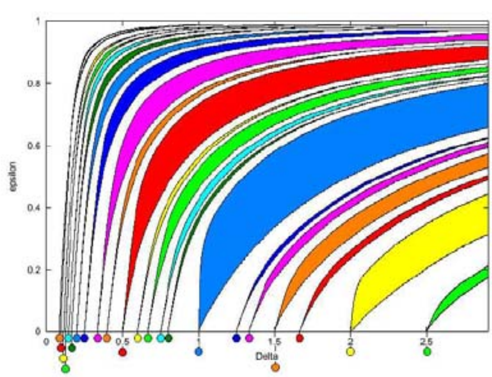
Kenesi, Molnár, Veres, Vattay SIGCOMM 2000





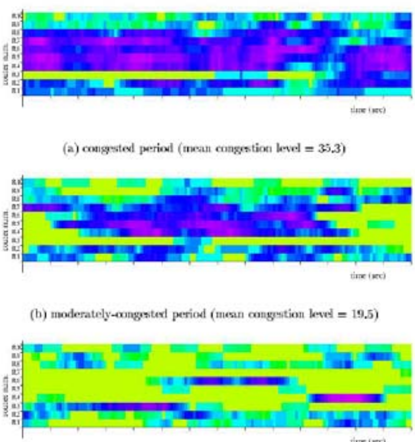
Mode locking structure of adaptation

Buchta & Vattay 2003



# Congestion propagation





(a) congested period (mean congestion level = 35.3)

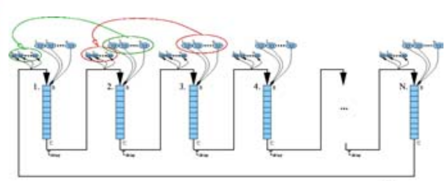
(b) moderately-congested period (mean congestion level = 19.5)

(c) non-congested period (mean congestion level = 7.5)

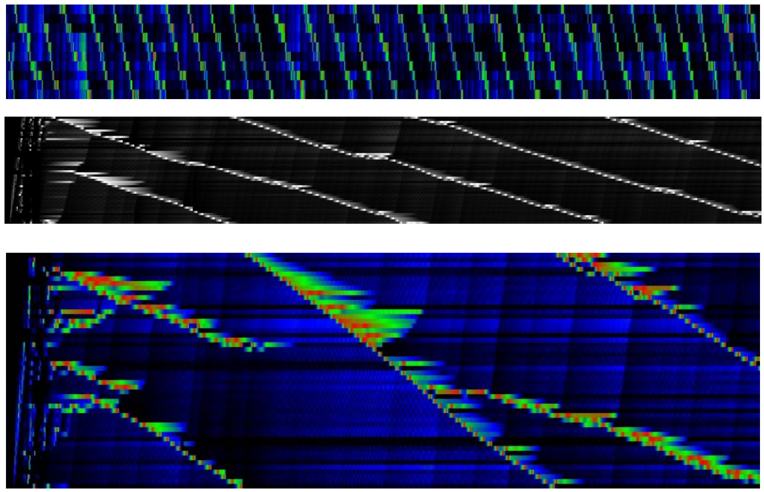
Figure 4.9: Spatio-temporal patterns of congestion

Fukuda &Takayasu 1999

Our model:



# Congestion propagation in the model



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